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Prediction of elastic modulus of high strength concrete by Gaussian Process Regression

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Abstract— Elastic moduli play a vital role in determining the deformation characteristics of concrete structures. This paper suggests a novel approach to predict the elastic modulus of high- strength concrete (HSC) using Gaussian Process Regression (GPR). GPR model is used to establish a relation between the modulus of elasticity and the compressive strength of HSC. GPR model is a non-parametric black-box model which searches for the relationship among measured data and estimates distributions over functions. GPR models are constructed and tested using available test data attained through existing literature studies. The first dataset used in this study is derived from experimental results. Out of a total of 87 cases of data, 79 are used for training and the rest are used to test the created model. The data are normalized between 0 and 1 prior to being used in the model. A subsequent parametric analysis is carried out to evaluate the sensitive variations of the elastic modulus corresponding to variations in the compressive strength. The GPR model accurately predicts the elastic modulus of concrete blocks considered. The output of the GP model is a normal distribution, expressed as mean and variance - the former represents the most likely output and the latter is a measure of its confidence. The results predicted are compared to those obtained from empirical results from buildings codes and various models – accurate results portray the strong potential of GPR, as a feasible and reliable tool, to predict elastic moduli of concrete.

Index Terms— Compressive strength, Elastic modulus, Gaussian Process Regression, High strength concrete, Normal distribution,Training data, Testing data

1 Introduction

ELASTIC modulus is a significant property of concrete which has been used to calculate deformation of structures [1]. Thence, making elastic moduli an imperative value to be predicted for different structural engineering problems. It is usually determined from the stress–strain graph of concrete, subjecting concrete cylinder under compression. Determining elastic modulus using conventional method engages complex testing steps like cyclic loading and strain measurements. This makes determination of elastic modulus, a complicated and time-consuming task compared with measuring compressive strength. As a result, many researchers and engineers have attempted to predict the elastic modulus using theoretical and empirical approaches. Elastic modulus has been predicted based on compressive strength (σc) of concrete. For example American [2], European [3], and Norwegian [4] committees recommended the following equations for HSC, respectively:

$$E_c = 3.32(f_c)^{1/2} + 6.9 \quad (1)$$
$$E_c = 10\,(f_c + 8)^{1/3} \quad (2)$$
$$E_c = 9.5\,(f_c)^{0.3} \quad (3)$$

Where, f_c is compressive strength (in MPa) and E_c is elastic modulus (in GPa).

Apart from empirical relations, prediction of elastic modulus have also been done by using Data Mining techniques such as Artificial Neural Network (ANN) and the fuzzy logic model [5,6]. Though, ANN has some shortcomings like slow convergence speed, poor generalising performance whilst fuzzy logic model faces problem during determining fuzzy rules. This paper presents Gaussian process (GP) model as an alternative for the determination of elastic modulus, overcoming many previously faced problems.

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2. Gaussian Process Regression

The GP model is a non-parametric black-box model which searches for the relationship among measured data. It differs from other black-box classification approaches since it does not try to approximate the modelled system by fitting constraints of particular basis functions. GP model uses kernel functions similar to Support Vector Machines (SVM) and Relevance Vector Machines (RVM) [7, 8]. It gives normally distributed output, articulated in terms of mean and variance. The former value represents the most likely output and the latter can be inferred as the measure of its confidence. The latter value is dependent on the amount and

quality of available identification data. Let us consider the following dataset:

$$\{(x_k, y_k)\}^{N}_{k=1},\ x \in R\ y \in R \quad (4)$$

Where, x is input variable, y is output variable, R is one-dimensional vector space and N is number of data.

This study uses f_c as input variable and E_c as output variable. So, $x = [f_c]$ and $y = [E_c]$. The above dataset have been drawn from the following noise process.

$$y_i = f(x_i) + \varepsilon,\ \varepsilon \sim N(0, \sigma^2) \quad (5)$$

For a given input x*, GPR defines a gaussian predictive distribution over the output y* with mean

$$\mu = K(x^*,x)(K(x, x) + \sigma^2 I)^{-1} y \quad (6)$$

Where K(x,x) is kernel function and I is identity matrix and variance is represented by following equation:

$$\Sigma = K(x^*, x^*) - \sigma^2 I - K(x^*, x)(K(x, x) + \sigma^2 I)^{-1} K(x, x^*) \quad (7)$$

A suitable covariance function and its parameter are required to develop the GPR model. For a fixed value of gaussian noise, GPR is trained by maximizing marginal likelihood.

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3 GPR FOR PREDICTION OF ELASTIC MODULUS OF HSC

As stated above, various methods have been proposed for the prediction of elastic modulus of concrete using compressive strength value. This study employs the use of Gaussian Process Regression (GPR) for prediction of modulus of elasticity (E_c). The experimental results for HSC are taken from Wee [9] and Gesoglu [10]. GPR uses 79 out of 87 cases for HSC in order to train the model while rest for testing the developed model. The data are normalized between 0 and 1 before being used in the model as following:

$$D_{norm} = (D - D_{min})/(D_{max} - D_{min}) \quad (8)$$

Where, D_{max} and D_{min} are the maximum and minimum values of input and output data.

Normalized values of E_c and F_c of training dataset are given as input in GPR model. The value of the co-relation coefficient 'R' obtained from GPR model is 0.77. Fig.1 shows performance of testing dataset in terms of R. This value is achieved at a radial basis function width 'σ' = 0.0001 and noise 'ε' = 4. The developed model is then tested using testing dataset (see table 1).

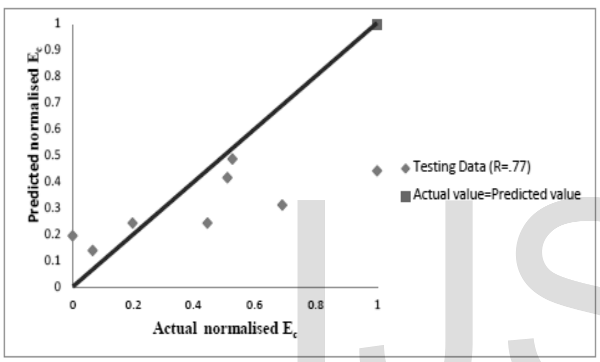

Fig. 1. Performance of the GPR model in the prediction of Ec

4 RESULTS AND DISCUSSIONS

Actual E_c values for testing dataset are compared with already existing models including the recently developed GPR model (see Table 1). The comparison has been shown in terms of errors (predicted values subtract measured values). In order to explicitly compare the magnitude of error, Root Mean Square Error (RMSE) is calculated. The RMSE value for GPR is shown in Fig. 2 along with the other developed models RMSE values. The model with the smallest RSME can be considered as the best model. From Fig. 2, we can infer that the GPR model predicts elastic modulus with much less error as compared to previously existing models. The study shows that the GPR has strong potential to predict accurate elastic modulus of HSC.

Table 1. Comparison of errors estimated by GPR and other models for testing dataset

F_c	E_c	ACI363	CEB	NS3473	Wee	Gesoglu	Regression	Fuzzy	ANN	SVM	GPR
69.7	41.5	-7.1	1.2	-7.5	0.4	-0.8	0.4	1.7	2.5	1.7	0.46
78.3	44.3	-8	0	-9.3	-0.4	-0.9	-0.4	0	0.4	-0.9	-0.21
82.6	44.2	-7.1	0.9	-8.4	0.4	0.4	0.9	0.9	1.3	0	-0.55
87.2	41.1	-3.3	4.5	-4.9	4.9	4.1	4.9	4.9	4.5	4.1	1.2
84.5	45.3	-7.7	0	-9.5	0	-0.5	0	0	0	-0.9	-2.2
77	47.2	-11.3	-3.3	-12.3	-3.8	-4.2	-3.3	-4.7	-2.4	-2.4	-3.39
86	43.8	-6.1	1.8	-7.4	1.3	2.2	2.2	0	1.8	1.3	-1.2
86	42.3	-4.7	3	-6.3	2.5	3.4	3.4	1.7	3.4	2.5	0.29

5 CONCLUSIONS

This study has investigated the use of GPR for prediction of elastic modulus of concrete from compressive strength. RMSE of the predicted results by GPR method is the smallest among all the prediction methods. Thus, GPR model can be regarded as a very efficient method to predict elastic modulus of HSC from their compressive strength value. Precision in prediction of elastic modulus increases with increase in the amount of data and number of regressors.

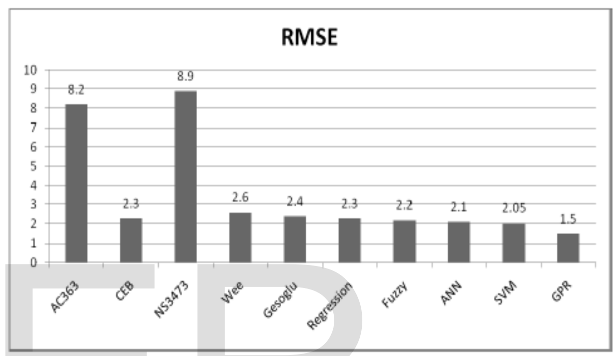

Fig. 2 Comparison between GPR and other models in terms of Root Mean Square Error

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